Δr	= step size in radial direction
$\Delta\theta$	= angular step size
$\Delta\tau$	= time step size
θ	= polar angle, spherical coordinate system
λ	= thermal conductivity
μ	= dynamic viscosity
ρ	= density
τ	= dimensionless time, $\tau = \alpha_2 t/a^2$
ψ	= stream function

Subscripts

i,k	= mesh point coordinates
0	= initial condition
1	= refers to disperse phase (interior of the droplet)
2	= refers to continuous phase (exterior of the droplet)
∞	= refers to large distance from the spherical particle

Superscripts

*	= asymptotic value (for $\tau \to \infty$)
—	= volume- or surface-averaged value

$j, j+1/2, j+1$ correspond to time τ, $(\tau + \Delta\tau/2)$, and $(\tau + \Delta\tau)$, respectively

LITERATURE CITED

Abramzon, B. M., V. Ya. Rivkind, and G. A. Fishbein, "Unsteady Mass Transfer Involving Heterogeneous Chemical Reaction in the Laminar Flow about a Sphere," *J. Eng. Phys. (USSR), (Inzh. Fiz. Zh.)*, **30**, (1), 73 (1976).

Abramzon, B. M., and G. A. Fishbein, "Some Convective Diffusion Problems to a Spherical Particle for $Pe \leq 1{,}000$," *J. Eng. Phys. (USSR), (Inzh-Fiz. Zh.)*, **30**, (6), 1053 (1977).

Brignell, A. S., "Solute Extraction from an Internally Circulating Spherical Liquid Drop," *Int. J. Heat Mass Transfer*, **18**, 61 (1975).

Brounshtein, B. I., A. S. Zheleznyak, and G. A. Fishbein, "Heat and Mass Transfer in Interaction of Spherical Drops and Gas Bubbles with a Liquid Flow," *Int. J. Heat Mass Transfer*, 13, 963 (1970).

Brounshtein, B. I., and G. A. Fishbein, *Hydrodynamics, Mass and Heat Transfer in a Dispersed System, Khimiya*, Leningrad in Russian, (1976).

Brown, A., "Diffusion of Heat from a Sphere to a Surrounding Medium," *Aust. J. Phys.*, 18, 483 (1965).

Carslaw, H. S., and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd ed., Oxford University Press, London (1959).

Chao, B. T., "Transient Heat and Mass Transfer of a Translating Droplet," *J. Heat Transfer*, 273 (1969).

Clift, R., J. R. Grace and M. E. Weber, *Bubbles, Drops and Particles*, Academic Press (1978).

Cooper, F., "Heat Transfer from a Sphere to an Infinite Medium," *Int. J. Heat Mass Transfer*, **26**, 991 (1977).

Elzinga, E. R., Jr., and J. T. Banchero, "Film Coefficients for Heat Transfer to Liquid Drops," *Chem. Eng. Prog.* Symp. Ser., **55**, 149 (1959).

Greskovich, E. J., P. Barton and R. E. Hersh, "Heat Transfer in Liquid-Liquid Spray Towers," *AIChE J.*, **13**, (6), 1160-1166 (1967).

Gröber, H., S. Erk, and U. Grigull, *Fundamentals of Heat Transfer*, McGraw-Hill, N.Y. (1961).

Head, H. N., and J. D. Hellums, "Heat Transport and Temperature Distributions in Large Single Drops at Low Reynolds Numbers: A New Experimental Technique," *AIChE J.*, **12**, (3), 553 (1966).

Heertjes, P. M., W. A. Holve and H. Talsma, "Mass Transfer between Isobutanol and Water in a Spray-Column," *Chem. Eng. Sci.*, 3, 122 (1954).

Johns, L. E., and R. B. Beckmann, "Mechanism of Dispersed-Phase Mass Transfer in Viscous, Single-Drop Extraction Systems," *AIChE J.*, **12**, (1), 10 (1966).

Konopliv, N., and E. M. Sparrow, "Unsteady Heat Transfer and Temperature for Stokesian Flow about a Sphere," *J. Heat Transfer*, 266 (1972).

Kronig, R., and J. C. Brink, "On the Theory of Extraction from Falling Droplets," *Appl. Sci. Res.*, **A2**, 142 (1950).

Levich, V. G., *Physicochemical Hydrodynamics*, Prentice-Hall, Englewood Cliffs, N.J. (1962).

Levich, V. G., V. S. Krylov, and V. P. Vorotilin, "On the Theory of Unsteady Diffusion from Moving Droplets," translation from *Doklady Akademii Nauk SSSR*, 161, 648 (1965).

Newman, A. B., "The Drying of Porous Solids: Diffusion Calculations," *Trans. Am. Inst. Chem. Eng.*, **27**, 310 (1931).

Pierce, R. D., O. E. Dwyer, and J. J. Martin, "Heat Transfer and Fluid Dynamics in Mercury-Water Spray Columns," *AIChE J.*, **5**, (2), 257 (1959).

Philip, J. R., "Transient Heat Conduction between a Sphere and a Surrounding Medium of Different Thermal Properties," *Aust. J. Phys.*, **17**, 423-30 (1964).

Richtmyer, R. D. and K. W. Morton, *Difference Methods for Initial Value Problems*, Interscience Publ., N.Y. (1967).

Rod, V., "Combination of Resistances to Mass Transfer between a Drop and an Ambient Liquid," *Chem. Eng. J.*, **15**, 179 (1978).

Sideman, S., "Direct Contact Heat Transfer between Immiscible Liquids," *Adv. Chem. Eng.*, **6**, 207-286 (1966).

Watada, H., A. E. Hamielec, and A. I. Johnson, "A Theoretical Study of Mass Transfer with Chemical Reaction in Drops," *Can. J. Chem. Eng.*, **48**, 255 (1970).

Wellek, R. M., W. V. Andoe, and R. J. Brunsen, "Mass Transfer with Chemical Reaction Inside Single Droplets and Gas Bubbles: Mathematical Mechanisms," *Can. J. Chem. Eng.*, **48**, 645 (1970).

Woo, S. E., and A. E. Hamielec, "A Numerical Method of Determining the Rate of Evaporation of Small Water Drops Falling at Terminal Velocity in Air," *J. Atmos. Sci.*, **28**, 1448 (1971).

Manuscript received August 14, 1978; revision received August 23, and accepted October 15, 1979.

Emulsion Phase Residence Time and Its Use in Heat Transfer Models in Fluidized Beds

TANER F. OZKAYNAK

and

JOHN C. CHEN

Department of Mechanical Engineering and Mechanics
Lehigh University
Bethlehem, Pennsylvania 18015

Heat transfer from a vertical tube in fluidized beds was investigated by measurement of emulsion packet residence times. The root-square-mean residence times were then used in a modified packet model to successfully predict effective heat transfer coefficients.

SCOPE

Many theoretical models have been developed for estimating the heat transfer coefficient to tubes submerged in fluidized beds. One of the oldest and most widely accepted is the packet renewal model originally developed by Mickley and Fairbanks (1955, 1961). In this, as in many of the other models, the heat transfer process is postulated to be governed by the residence time of the emulsion phase (packets) on the heat transfer sur-

face. To date, use of these models has been hampered by a lack of information regarding the actual residence times that exist in fluidized beds.

Ozkaynak and Chen (1974, 1978) reported measurements of the average residence times on a tube submerged in fluidized beds, obtained by a fast response capacitance probe. This technique was used in the present study to experimentally investigate the detailed statistical variations of packet residence times in air fluidized beds of glass spheres. This information permitted the direct calculation of heat transfer coefficients by the packet renewal models. Independent measurements of heat transfer, under identical fluidizing conditions, were then obtained to provide a direct comparison with theory.

CONCLUSION AND SIGNIFICANCE

The mechanism of heat transfer from a centrally located vertical tube, submerged in air fluidized beds of glass spheres, was investigated experimentally. From fast response capacitance probes mounted flush within the tube, the residence times of emulsion (packet) phase and void (bubble) phase at the surface of the submerged tube were determined at air flow rates up to fifteen times minimum fluidization flow rate. Diameter of the cylindrical bed and the submerged tube were 14.0 cm and 12.7 mm, respectively. Four different classifications of glass spheres were used in the fluidized beds, with diameters ranging from 136 to 610 μm and solid densities ranging from 2.47 to 4.49 g/cm^3.

For any given fluidization state, the local residence time of the emulsion phase on the tube surface was found to have a characteristic log normal distribution. Arithmetic average residence times were found to be in the range of 0.07 to 1.13 s. The fraction of time when a local area of the tube was covered by emulsion packet was in the range of 90 to 40%.

The heat transfer coefficients were measured for the same conditions used for the fluid mechanic experiments. Data were obtained to show the effects of elevation, particle size, particle density and flow rate on the heat transfer coefficients.

Validity of Mickley and Fairbank's packet model for heat transfer was investigated. The model was used in its original form with the help of the statistical information obtained in this study about the emulsions (packets) and voids. It was found that the agreement between this theory and the experiments is very good for small particles. However, the model overestimated the heat transfer coefficients for large particles.

A modified packet model was introduced which accounts for the change of void fraction in a packet near the surface of the heater. This modified model successfully predicted heat transfer for both small and large particles with different physical and thermal properties.

EXPERIMENT

The experiments were performed in a fluidized bed facility described in a previous publication (Ozkaynak and Chen, 1974). The cylindrical bed was of 14.0 cm I.D. and 1.22 m height. Measurements were obtained on a vertical tube, 12.7 mm diameter, located at the bed axis. Two types of data were obtained: capacitance probe measurements of emulsion contact times and separate measurements of the local heat transfer coefficients. All measurements were obtained at the surface of the tube at an instrumented section.

The capacitance probe and associated instrumentation for contact measurements were described in a previous study (Ozkaynak and Chen, 1978), and the reader is referred to that paper for details. Briefly, the technique utilized miniature capacitance sensors mounted flush in the surface of the test tube. A 10 MHz bridge with associated oscillograph recorder permitted direct determination of local tube contact with either emulsion (packet) phase or a void (bubble) phase. Residence times were determined from elapsed times between successive bubbles.

A heater was developed to measure the local, time averaged, heat transfer coefficients. It consisted of a copper tube with a cylindrical cartridge heater and two thermocouples to measure the wall temperatures, as shown in Figure 1. The heating element was a 200 W, 10.16 cm long and 6.35 mm O.D. cartridge heater. It was placed coaxially into a thick walled 12.7 cm long and 1.27 cm O.D. copper tube. The inside diameter of the copper tube was drilled about 0.381 mm larger than the cartridge heater and the gap was filled with copper powder. To prevent axial conduction, teflon plugs were placed at both ends of the heater. Two thermocouples were placed on the surface of the heater to measure wall temperatures. One thermocouple junction was located at the center of the 10.16 cm heated length. The other thermocouple junction was placed just at the end of the heated length. Both of the thermocouples were silver soldered into grooves in the copper tube.

The bed temperature was measured by an immersed thermocouple at the same level as the center of the heater, 2.5 cm radially outward from the tube surface. The heat release rate of the heater was determined by measurement of the power dissipated. As in the capacitance measurements, heat transfer measurements were made at three different elevations: 7.62, 15.24 and 22.86 cm above the distributor plate.

Four types of glass spheres were used as the bed particles. All test particles were selected for high sphericity, and each type had a controlled narrow size range. Dimensions and properties for these particles are given in Table 1. In all tests, the particles were packed to a static bed height of 30.5 cm.

THEORY

Mickley and Fairbanks (1955, 1961) postulated that heat transfer takes place by unsteady state diffusion of heat from the surface into packets of solid particles and interstitial gas (emulsion). The governing equation for the heat transfer mechanism is a one-dimensional conduction equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha_p}\frac{\partial T}{\partial \theta} \tag{1}$$

with the boundary conditions

$$T(x,\,0) = T_b$$
$$T(0,\,\theta) = T_w$$
$$T(\infty,\,\theta) = T_b$$

The predicted time-mean local heat transfer coefficient is, then

$$\bar{h}_1 = (1 - f_0)\frac{2}{\sqrt{\pi}}\sqrt{k_e\rho_e c_e}\frac{1}{\sqrt{\bar{\theta}'_e}} + f_0 \cdot h_0 \tag{2}$$

Here

$$\bar{\theta}'_e = \left[\frac{\sum_{n=1}^{N} \theta_{e_n}}{\sum_{n=1}^{N} \sqrt{\theta_{e_n}}}\right]^2 \tag{3}$$

is called root-square average emulsion (packet) residence time.

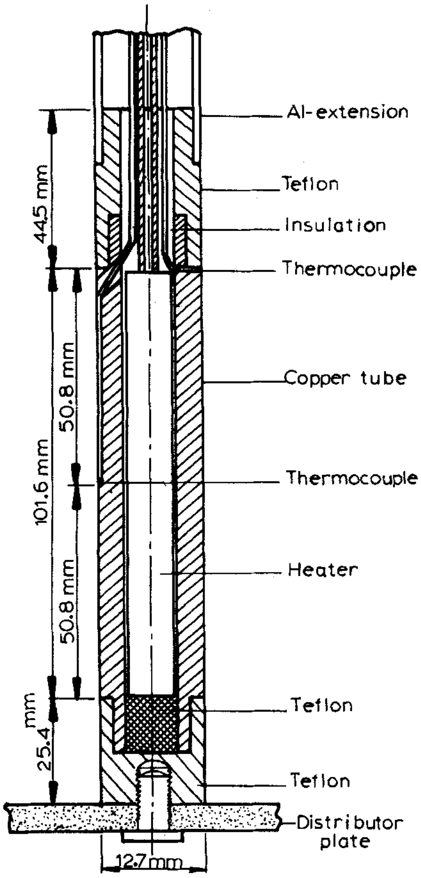

Figure 1. The heater used in the heat transfer experiments.

Equation (2) was first proposed by Mickley and Fairbanks (1955, 1961) but has not been used in this form owing to lack of information about $\bar{\theta}_e'$ and f_0.

In this study, density of an emulsion packet (ρ_e) was taken to be equal to the density of a loosely packed static bed of particles. Void fraction was measured experimentally, and the packet density was calculated by

$$\rho_e = (1 - \epsilon_1)\rho_s \tag{4}$$

Since the fluidizing medium is gas, its heat capacity can be neglected relative to that of solids. Heat capacity of packets (c_e) are assumed to be equal to the solid heat capacity (c_s).

In the theoretical calculations of the heat transfer coefficient in this study, emulsion packet conductivity was taken to be

$$k_e = k_{es} + k_{ef} \tag{5}$$

The effective thermal conductivity of packed beds filled with stagnant fluid has been correlated by Yagi and Kunii (1957), Kunii and Smith (1960) and Koya and Kunii (1972). Another correlation was proposed by Baskakov (Davidson and Harrison, 1971). Comparison of the effective conductivities calculated with these two equations shows good agreement for the void fractions encountered. Kunii's correlation was subsequently used in this study. Conductivity due to flow (k_{ef}) was calculated by the correlation proposed by Ranz (1952).

RESULTS OF RESIDENCE TIME EXPERIMENTS

A characteristic variation of average residence time of emulsion phase (packets) $\bar{\theta}_e'$ with flow rate for different size particles is shown in Figure 2. It is clearly seen that the increasing particle diameter increases the average residence times of the emulsion phase (packets) for a given excess air flow rate ($V - V_{mf}$). The residence time is very long at air flow rates close to minimum fluidization and decreases rapidly with increasing flow rate, reaching a constant value asymptotically around 0.20 s.

In addition to data on average residence times, measurements were obtained of the statistical variations in residence times. The probability distribution function of residence times $p(\theta)$ and the cumulative distribution function $F(\theta)$ was obtained by dividing the range for θ in an appropriate number of intervals and tabulating the number of observation and the percentage of data in each interval (Bendat et al., 1966).

Some typical residence time density functions are shown by the points in Figure 3. It is seen that the distributions are skewed, with maxima near short periods and a long asymptotic decay tail at long periods. The distributions are more highly peaked at high flow rates. Some typical residence time, cumulative distribution data are shown by the points in Figure 4. It is clearly seen that the short residence times dominate at high flow rates and long residence times at low flow rates.

Two theoretical distribution functions were considered for the measured distributions: the gamma distribution as proposed by Ziegler et al. (1964) and the log-normal distribution.

The log-normal distribution, in its simplest form, may be defined as the distribution of a variate whose logarithm obeys the normal law of probability (Lipson et al., 1973; Aitchison et al., 1963). If z is a random variable which has a log-normal distribution, then $y = \log z$ will have a normal distribution. Mathematically, θ has a log-normal distribution if

$$p(\theta) = \frac{1}{s_l\sqrt{2\pi}} \exp\left[\frac{-(\theta_l - \mu_l)^2}{2s_l^2}\right] \quad -\infty \leq \theta_l \leq \infty \tag{6}$$

where

TABLE 1. PROPERTIES OF SOLID PARTICLES

Quality of particles	GT-1 tech. qual. glass beads	GT-2 tech. qual. glass beads	GT-3 tech. qual. glass beads	GH-4 high density glass beads
Mean particle diameter (μm)	136	245	610	241
U.S. sieve size	100-140	50-70	25-35	50-70
Particle thermal conductivity (W/mK)	0.89	0.89	0.89	1.00
Particle heat capacity (J/kg·K)	753.6	753.6	753.6	439.6
Particle density (g/cm^3)	2.47	2.47	2.47	4.49
Bed void fraction				
Loose packed	0.401	0.390	0.386	0.389
Dense packed	0.373	0.352	0.361	0.356
Bed density (g/cm^3)				
Loose packed	1.48	1.51	1.52	2.74
Dense packed	1.55	1.60	1.58	2.89
Packet conductivity (stagnant fluid) (W/mK)	0.176	0.186	0.190	0.200
Minimum fluidization velocity (cm/s)	3.17	5.88	23.26	9.24

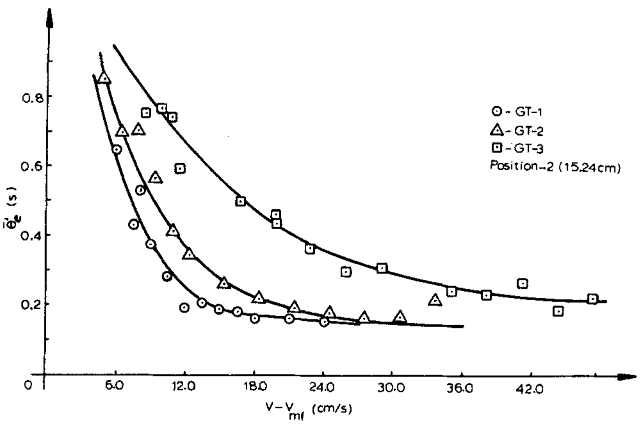

Figure 2. Variation of residence time with particle size.

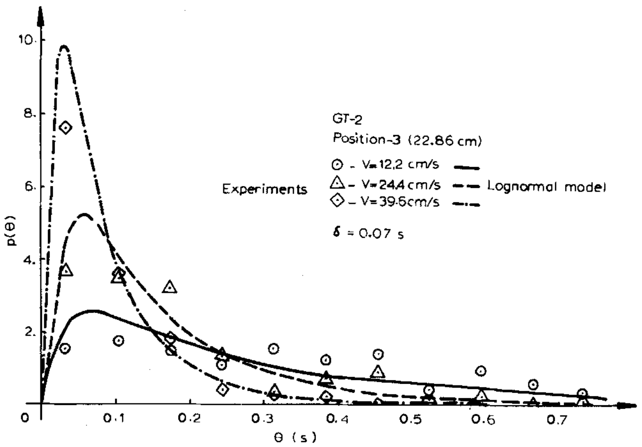

Figure 3. Comparison of lognormal density function with density histograms.

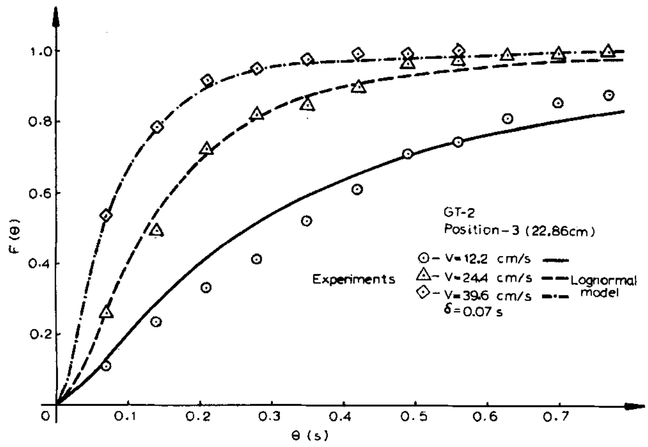

Figure 4. Comparison of lognormal cumulative distribution with experiments.

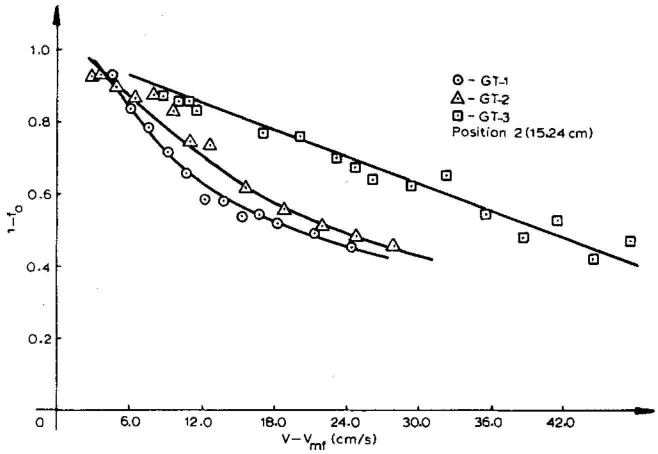

Figure 5. Effects of particle size on $(1 - f_0)$.

$$\theta_l = \log \theta$$

$$\mu_l = \int_{-\infty}^{+\infty} \theta_l p(\theta) d\theta_l \qquad (7)$$

$$s_l^2 = \int_{-\infty}^{+\infty} (\theta_l - \mu_l)^2 \cdot p(\theta) d\theta_l$$

As estimates for log-normal population mean value (μ_l) and the log-normal population variance (s_l^2), the log sample mean was calculated by

$$\bar{\theta}_l = \frac{1}{N} \sum_{n=1}^{N} \log \theta_{e_n} \qquad (8)$$

The log sample variance was calculated by

$$\sigma_l^2 = \sum_{n=1}^{N} \frac{(\log \theta_{e_n} - \bar{\theta}_l)^2}{N-1} \qquad (9)$$

The values for $\bar{\theta}_l$ and σ_l^2 were calculated for each run and were given in Ozkaynak and Chen (1974). The experimental probability density functions are compared to log-normal distribution function in Figure 3 for medium size (GT-2) particles. The cumulative log-normal distribution was also calculated for the same case and shown in Figure 4 with the experimental distribution function calculated from the data. It is seen that the log-normal distribution does fit the measured residence time distributions well for the conditions of this experiment.

Another result of the experiment is the measurement of the fraction of the total time that the surface is occupied by void phase (f_0) or emulsion phase ($1 - f_0$). A characteristic variation of ($1 - f_0$) with the flow rate for different size particles is shown in Figure 5. It is clearly seen that increasing particle diameter increases the emulsion phase occupancy for a given excess flow rate ($V - V_{mf}$). For flow rates close to minimum fluidization, the surface is mostly covered by the emulsion phase (packets), but as the flow increases, emulsion occupancy decreases while the fraction of the total time that the surface is occupied by voids increases. These results are generally consistent with those reported by Mickley and Fairbanks (1961).

RESULTS OF HEAT TRANSFER EXPERIMENTS

Some results of heat transfer experiments are shown for GH-4 particles as a function of superficial velocity in Figure 6 for three

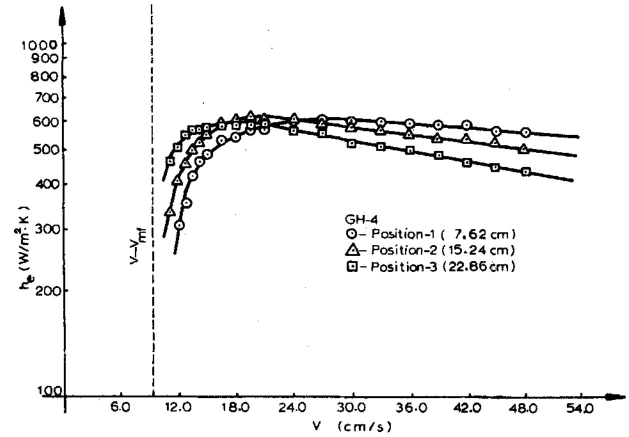

Figure 6. Effect of location on heat transfer coefficient.

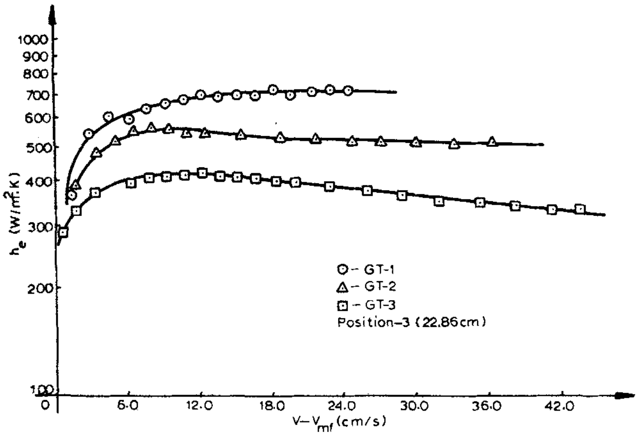

Figure 7. Effect of particle size on heat transfer coefficient.

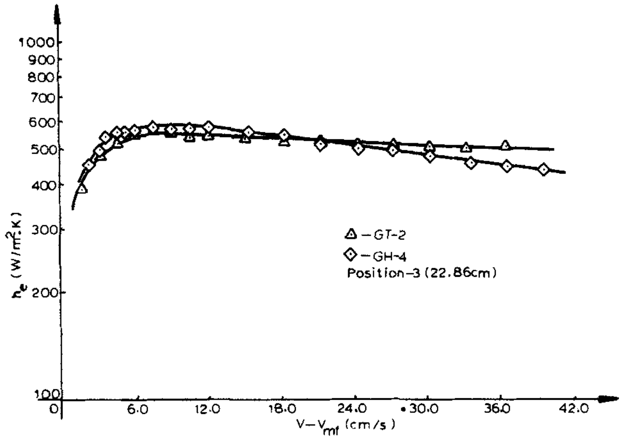

Figure 8. Effect of particle density and thermal properties on the heat transfer coefficient.

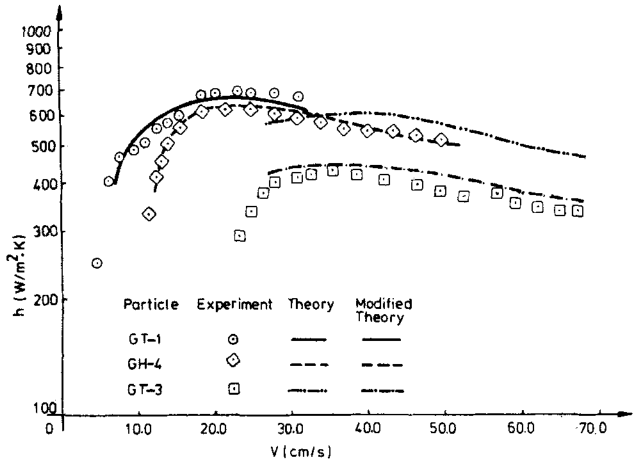

Figure 9. Comparison of experimental and theoretical heat transfer coefficients.

different elevations. At low flow rates, bubbles are infrequent so that packets tend to have long residence times at the heat transfer surface, leading to low heat transfer coefficients. This is particularly true at low elevations where the bubbles are small and can easily miss the heat transfer tube. In this situation, coalescence to give larger bubbles tends to cause a higher frequency of packet replacement (and higher heat transfer coefficients) at higher elevations within the bed. At higher flow rates, bubbles are much more frequent, so that packet residence times are short and good heat transfer is obtained. In this situation, bubble coalescence at the upper elevations tends to reduce packet replacement (due to diminishing frequency) and cause a slight decrease in heat transfer coefficient with increasing elevation.

Effect of the particle size on heat transfer coefficient is seen in Figure 7. As expected from residence time measurements (Figure 2), small particles have higher heat transfer coefficients for the same excess velocity, corresponding to their shorter residence times as seen from Equation (2).

GT-2 and GH-4 particles have about the same diameter but have different physical and thermal properties. In the packet model, the heat transfer coefficient is directly proportional to packet conductivity and inversely proportional to the square root of packet diffusivity. For these particles, this ratio $(k_e/\sqrt{\alpha_e})$ is roughly equal (see Table 1). Thus, the only difference in heat transfer coefficient would come from the fluidization characteristics of the particles. It was shown by Ozkaynak and Chen (1978) that these particles have the same residence time and same frequency of replacement for the same excess velocity, so that the heat transfer coefficients should be about the same. This is seen to be confirmed by the data in Figure 8.

COMPARISON OF EXPERIMENTAL HEAT TRANSFER COEFFICIENTS WITH THE PACKET MODEL

The fraction of the total time that the surface is covered by emulsions $(1 - f_0)$ and by voids (f_0) and the root-square average residence time $(\bar{\theta}_e')$ were measured, and the results are shown above and in Ozkaynak and Chen (1974, 1978). Calculation of emulsion phase properties k_e, ρ_e and c_e has also been explained above. Now we are in a position to check the validity of the packet model by comparing the heat transfer coefficient obtained from Equation (2) with the independent heat transfer measurements. For the conditions of this study, the heat transfer to the voids is very small (<2%) compared to heat transfer to emulsion (packet). For this reason, heat transfer to the voids was ignored in the calculations.

Some of the results for GT-1, GH-4 and GT-3 particles are shown in Figure 9. As is seen from this figure, the agreement between the theory and the experiment is very good for small and medium particles (GT-1 and GH-4). The agreement is not good for the larger GT-3 particles.

An explanation can be found if the temperature profiles of packets at different residence times are examined. Even for low residence times, the temperature wave penetrates at least to a depth of 5 or 6 diam for small (GT-1) particles and 3 or 4 diam for medium (GH-4, GT-2) particles. However, the penetration depth is only about 1 (or less than 1) diam for large (GT-3) particles. At depths of below 1 diam, the local packet properties (that is, void fraction, effective conductivity, etc.) may be significantly different from bulk properties.

Kimura et al. (1955), Schwartz (1958) and Kubie and Broughton (1975) showed that the void fraction near the wall is larger than the void fraction in the core portion of a packed bed. For distances greater than 1 particle diam from the wall, the packed bed has the bulk core void fraction, but for distances less than this, local void fraction rapidly increases approaching the wall. If the void fraction of the packed bed at the core is defined as ϵ_1, the following equations can be derived for the distribution of void fraction near the wall from Kimura's work (Yagi and Kunii, 1961):

$$\epsilon = \epsilon_1 - 0.304 \log_{10}\left(\frac{x}{D_p}\right) \text{ for } \frac{x}{D_p} < 1 \qquad (10)$$

$$\epsilon = \epsilon_1 \text{ for } \frac{x}{D_p} \geq 1 \qquad (11)$$

Thus, the packet model for heat transfer in fluidized beds gives higher values for the heat transfer coefficient than the experimental results for large particles because it overestimates the effective packet conductivity by underestimating its effective void fraction. Yasutomi and Yokota (1976) took account of this fact by regarding the emulsion phase as composite solids consisting of two layers. Kubie and Broughton (1974) modified the packed model to allow for property variations in the packet in the region of the surface. They solved their governing equations numerically by using a finite difference method. In this study a method is proposed which enables the direct calculation of heat transfer coefficient by a modification of the packet theory.

If the penetration depth of the temperature gradient for each residence time is defined as the distance from the wall to the place, where

$$\frac{T_w - T}{T_w - T_b} = 0.9 \tag{12}$$

then this depth can be used as a criterion for finding the void fraction in the effective region. It is proposed that the packet has the effective void fraction of a packed bed at a distance half of penetration depth away from the wall. This means that if the penetration depth is more than 2 particle diam, the packet has the bulk void fraction of a packed bed by Equation (11). If the penetration depth is less than 2 particle diam, then the effective void fraction is found by Equation (10).

From the arithmetic average residence times found from experiments, penetration depths were obtained, and the void fractions were calculated at a distance equal to half of the penetration depth by Equations (10) or (11). Using these effective void fractions, one can then calculate local emulsion conductivities and corresponding heat transfer coefficients by the packet model.

The heat transfer coefficients calculated by this modified model are also shown in Figure 9 for the large particles. Good agreement with the experimental heat transfer coefficient is now obtained. If this modified packet model is applied to the other (smaller) particles (GT-1, GT-2 and GH-4), it is seen that the penetration depth is always greater than 2 particle diam, so that no correction is required for void fraction.

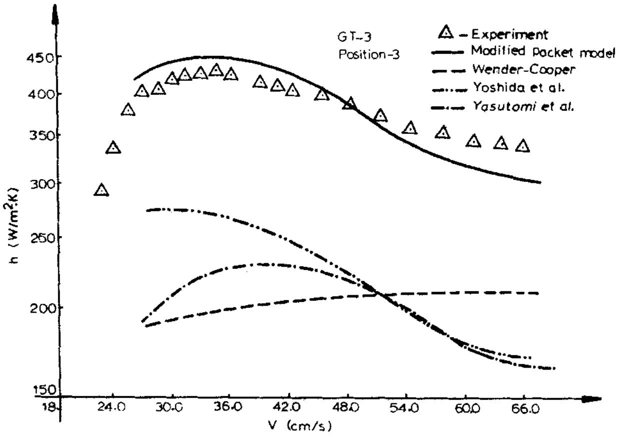

Figure 10. Comparison of different models and correlations.

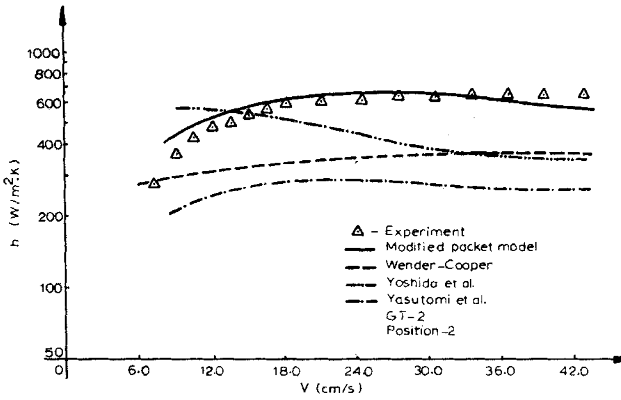

Figure 11. Comparison of different models and correlations.

COMPARISON WITH OTHER MODELS AND CORRELATIONS

A number of different correlations have been proposed for heat transfer from fluidized beds to immersed tubes. The difference between values of h calculated by the various correlations can be greater than an order of magnitude.

One of the most widely used correlations is the one given by Wender and Cooper (1958):

$$\frac{hD_p}{k_g} = 0.033\,(1-\epsilon)\left(\frac{c_s}{c_g}\right)^{0.8}\left(\frac{\rho_s}{\rho_g}\right)^{0.66}\left(\frac{D_p \cdot G}{\mu_g}\right)^{0.23}\left(\frac{c_g \cdot \rho_g}{k_g}\right)^{0.43} \tag{13}$$

The heat transfer coefficients calculated by this empirical correlation and by the models of Yoshida et al. (1969) and Yasutomi et al. (1976) are compared with the experimental results and the modified packet model in Figures 10 and 11. For those models requiring residence times, the measured residence time data were used. It is seen that for large GT-3 particles, the published models all underpredicted the heat transfer coefficients. For medium size GT-2 particles, the two-layer emulsion model of Yasutomi et al. predicted the heat transfer coefficient with reasonable agreement for low superficial velocities. All the published correlations again underpredicted the heat transfer coefficient at moderate to higher flow rates.

In contrast, it is seen that the modified packet theory, using measured residence time data, gave very good agreement with the experimental heat transfer coefficients for both small and large particles (GT-2 and GT-3, respectively).

SUMMARY

Heat transfer and fluid mechanic characteristics of fluidized beds, and their dependence on the properties of the particles and operating conditions, were investigated.

Residence time of emulsion phase on the heat transfer surface was measured directly. Probability density and cumulative distribution functions of the residence times were obtained. Gamma and log-normal distributions were found to give good representation of the data. Experimental information was obtained about the fraction of the total time that the surface is covered by emulsions.

Effect of particle size, elevation and flow rate on heat transfer coefficient were measured with a specially designed heater.

Validity of packet model of heat transfer was investigated. With the measured residence data, it was found that the model was satisfactory for small particles of different physical and thermal properties but overpredicted the heat transfer coefficients for large particles.

A modified packet model was introduced which accounts for the change of void fraction near the surface of packets. This modified packet model was found to successfully predict heat transfer coefficients for both small and large particles and for particles with different physical and thermal properties.

NOTATION

c_e = emulsion heat capacity
c_g = gas heat capacity
c_s = solid heat capacity
D_p = particle diameter
e = 2.7182818
$F(\theta)$ = cumulative distribution function
f_0 = fraction of the total time that the surface is covered by the voids
G = mass flow rate of gas
$\underline{h}_e$ = experimental heat transfer coefficient
$\overline{h}_1$ = time-mean local heat transfer coefficient
h_0 = heat transfer coefficient to a void
k_e = thermal conductivity of the emulsion phase
k_{ef} = effective thermal conductivity of a packed bed due to

fluid flow

k_{es} = effective thermal conductivity of a packed bed in the absence of fluid flow
k_g = thermal conductivity of gas
log = natural logarithm
$\log_{10}$ = logarithm to base 10
N = number of samples
$p(\theta)$ = probability density function
s_l = log population standard deviation
T = temperature
T_b = bulk temperature of the bed
T_w = wall temperature of the heater
V = superficial air velocity
V_{mf} = minimum fluidization velocity
x = distance from heat transfer surface

Greek Letters

δ = interval length
ϵ = void fraction (effective)
ϵ_1 = void fraction of a loosely packed bed
ϵ_2 = void fraction of a densely packed bed
θ = time
θ_e = emulsion (packet) residence time
$\bar{\theta}_e'$ = root-square average emulsion (packet) residence time by Equation (3)
$\bar{\theta}_l$ = log sample mean of residence times
μ_g = viscosity of gas
π = 3.1415926
ρ_e = emulsion (packet) density
ρ_g = density of gas
ρ_s = solid density
σ_l = log sample standard deviation

LITERATURE CITED

Aitchison, J., and J. A. C. Brown, *The Lognormal Distribution*, Cambridge at the University Press, England (1963).

Davidson, J. F., and D. Harrison, *Fluidization*, Academic Press, New York (1971).

Bendat, J. S., and A. G. Piersol, *Measurement and Analysis of Random Data*, J. Wiley, New York (1966).

Kimura, M., et al., *Chem. Eng. (Japan)*, **19**, 397 (1955).

Koya, T., and D. Kunii, "Measurement of Thermal Conductivities of Solid Particles in Packed Beds," *Heat Transfer, Japanese Research*, **1**, 2 (1972).

Kubie, J., and J. Broughton, "A Model of Heat Transfer in Gas Fluidized Beds," *Int. J. Heat Mass Transfer*, **18**, 289 (1975).

Kunii, J. M., and D. Smith, "Heat Transfer Characteristics of Porous Rocks," *AIChE J.*, **6**, 71 (1960).

Lipson, C., and N. J. Sheth, *Statistical Design and Analysis of Engineering Experiments*, McGraw Hill, New York (1973).

Mickley, H. S., and D. F. Fairbanks, "Mechanism of Heat Transfer to Fluidized Beds," *AIChE J.*, **1**, 376 (1955).

————, and R. D. Hawthorn, *Chem. Eng. Progr. Symposium Ser. No. 32*, **51** (1961).

Ozkaynak, T. F., and J. C. Chen, "Packet Residence Time and its Relation with the Heat Transfer Coefficient in Fluidized Beds," *Thermosciences*, TS-742, Lehigh Univ., Pa. (1974).

————, "Average Residence Times of Emulsion and Void Phases at the Surface of Heat Transfer Tubes in Fluidized Beds," *AIChE Symposium Ser. No. 174*, **74**, 334 (1978).

Ranz, W. E., "Friction and Transfer Coefficients for Simple Particles and Packed Beds," *Chem. Eng. Progr.*, **48**, 247 (1952).

Schwartz, C. E., and J. M. Smith, "Flow Distribution in Packed Beds," *Ind. Chem. Eng.*, **45**, 1209 (1953).

Wender, L., and G. T. Cooper, "Heat Transfer Between Fluidized-Solid Beds and Boundary Surfaces—Correlation of Data," *AIChE J.*, **4**, 15 (1958).

Yagi, S. and D. Kunii, "Studies on Effective Thermal Conductivities in Packed Beds," *ibid.*, **3**, 373 (1957).

————, and N. Wakao, "Radially Effective Thermal Conductivities in Packed Beds," *Int. Dev. Heat Transfer, ASME*, **4**, 742 (1961).

Yagi, S., and D. Kunii, "Studies on Heat Transfer in Packed Beds," *ibid.*, 750 (1961).

Yasutomi, T., and T. Yokota, "Heat Transfer from the Surface of a Body Fixed in a Fluidized Bed," *Kagaku Koguku Ronbunshu*, **2**, 205 (1976).

Yoshida, K., D. Kunii, and O. Levenspiel, "Heat Transfer Mechanism Between Wall Surface and Fluidized Bed," *Int. J. Heat Mass Transfer*, **12**, 529 (1969).

Ziegler, E. N., L. B. Koppel and W. T. Brazelton, "Effects of Solid Thermal Properties on Heat Transfer to Gas Fluidized Beds," *Ind. Eng. Chem. Fundamentals*, **3**, 324 (1964).

Manuscript received May 4, 1979; revision received November 7, and accepted December 21, 1979.

The Critical Velocity in Pipeline Flow of Slurries

ANIL R. OROSKAR

Department of Chemical Engineering
Syracuse University
Syracuse, New York 13210

and

RAFFI M. TURIAN

Department of Chemical Engineering
Texas Tech University
Lubbock, Texas 79409

In slurry transport, the critical velocity is defined as the minimum velocity demarcating flows in which the solids form a bed at the bottom of the pipe (bed load flows) from fully suspended flows. An analysis based on balancing the energy required to suspend the particles with that derived from dissipation of an appropriate fraction of the turbulent eddies is used to develop a correlation for prediction of the critical velocity. Comparison of the results with available experimental critical velocity data, relating to a rather wide variety of slurry systems, confirms that the present correlation does a superior job of prediction than all previously proposed critical velocity correlations.

SCOPE

The principal objective of this work was to develop an analytical procedure for predicting the critical velocity for slurry transport in pipelines. The prediction of the critical velocity, defined as the minimum velocity demarcating flows in which the solids form a bed at the bottom of the pipe from fully suspended flows, is an eminently practical problem in itself. However, the analytical approach described here derives its broader relevance from the fact that it constitutes an attempt to establish a better understanding of the complex problem of slurry transport.

Correspondence concerning this work should be addressed to Raffi M. Turian. Anil R. Oroskar is now in the Department of Chemical Engineering, University of Wisconsin, Madison, Wisconsin 53706.

0001 - 1541 - 80 - 3462 - 0550 - $01.05